

# COMMONWEALTH OF AUSTRALIA

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Family Name	
Given Names	
Student Number	<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>
Teaching Period	Semester 2, 2016

FINAL EXAMINATION	DURATION				
STA101 – Statistics 1	<table> <tr> <td>Reading Time:</td><td>10 minutes</td></tr> <tr> <td>Writing Time:</td><td>180 minutes</td></tr> </table>	Reading Time:	10 minutes	Writing Time:	180 minutes
Reading Time:	10 minutes				
Writing Time:	180 minutes				

### INSTRUCTIONS TO CANDIDATES

None.

### EXAM CONDITIONS

**You may begin writing from the commencement of the examination session.** The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
none	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s, Tables

**THIS EXAMINATION IS PRINTED  
DOUBLE-SIDED.**

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**Question 1 (11 marks)**

Table 1 gives scores attained by the 25 members of a local high school graduating class.

- a) Draw a dotplot for the given scores. (5 marks)  
 b) Determine  $P_5$ ,  $P_{10}$ ,  $P_{20}$ ,  $P_{99}$ ,  $P_{90}$  and  $P_{80}$  for the given scores. (6 marks)

Table 1.

21	24	23	17	31	19	19	20	19	25	17	23	16
21	20	28	25	25	21	14	19	17	18	28	20	

**Question 2 (15 marks)**

In Table 2, the number of hours studied,  $x$ , is compared to the exam grade received,  $y$ .

- a) Find the equation for the line of best fit. (7 marks)  
 b) Draw scatter diagram of the given data. (3 marks)  
 c) Draw the line of best fit on the scatter diagram of the given data. (3 marks)  
 d) Based on the answers provided for parts (a), (b) and (c), does it pay to study for an exam? Explain. (2 marks)

Table 2.

$x$	2	5	1	4	2
$y$	80	80	70	90	60

**Question 3 (12 marks)**

In an aquatic centre, swim lessons ranging from Red Cross Level 2, Fundamental Aquatic Skills through Red Cross Level 6, Swimming and Skill Proficiency, are offered during two sessions. A program coordinator is going to randomly select one swimmer to be interviewed for a local television spot on the centre and its swim program. Based on the information given in Table 3, what is the probability that selected swimmer is in the following?

Table 3.

Level	Number of People in 10 a.m. Class	Number of People in 11 a.m. Class
2	12	12
3	15	10
4	8	8
5	2	0
6	2	0

- a) A level 3 class. (2 marks)
- b) The 10 a.m. class. (2 marks)
- c) A level 2 class, given that is 10 a.m. session. (4 marks)
- d) The 11 a.m. session, given that it is Level 6 class. (4 marks)

#### Question 4 (15 marks)

According to a statistics obtained from an organisation, the distribution of ages for licensed drivers has a mean of 47.5 years and a standard deviation of 16.6 years. Assuming the distribution of ages is normally distributed, what percentage of the drivers are:

- a) Between the ages 17 and 22? (3 marks)
- b) Younger than 25 years of age? (3 marks)
- c) Older than 21 years of age? (3 marks)
- d) Between the ages of 48 and 68? (3 marks)
- e) Older than 75 years of age? (3 marks)

#### Question 5 (25 marks)

The local bakery bakes more than a thousand loaves of bread daily, and the weights of these loaves vary. The mean weight is 482 grams. Assume that the standard deviation of the weights is 18 grams and that a sample of 40 loaves is to be randomly selected.

- a) This sample of 40 has a mean value of  $\bar{x}$ , which belongs to a sampling distribution. Find the shape of this sampling distribution. (1 marks)
- b) Find the mean of this sampling distribution. (1 marks)
- c) Find the standard error of this sampling distribution. (2 marks)
- d) What is the probability that this sample mean will be between 475 and 495? (7 marks)
- e) What is the probability that the sample mean will have a value less than 478? (7 marks)
- f) What is the probability that the sample mean will be within 5 grams of the mean? (7 marks)

#### Question 6 (12 marks)

Find the test statistic  $z^*$  and the p-value for each of the following situations.

- a)  $H_0: \mu=22.5$ ,  $H_a: \mu>22.5$ ;  $\bar{x}=24.5$ ,  $\sigma=6$ ,  $n=36$  (4 marks)
- b)  $H_0: \mu=200$ ,  $H_a: \mu<200$ ;  $\bar{x}=192.5$ ,  $\sigma=40$ ,  $n=50$  (4 marks)
- c)  $H_0: \mu=12.4$ ,  $H_a: \mu\neq 12.4$ ;  $\bar{x}=11.52$ ,  $\sigma=2.2$ ,  $n=16$  (4 marks)

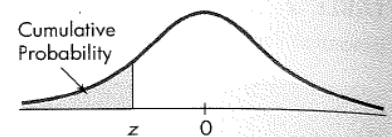
#### Question 7 (10 marks)

Determine the p-value for the following hypothesis tests involving Student's t-distribution with 10 degrees of freedom.

- a)  $H_0: \mu=15.5$ ,  $H_a: \mu<15.5$ ,  $t^*=-2.01$  (5 marks)
- b)  $H_0: \mu=15.5$ ,  $H_a: \mu\neq 15.5$ ,  $t^*=-2.01$  (5 marks)

**TABLE 3****Cumulative Areas of the Standard Normal Distribution**

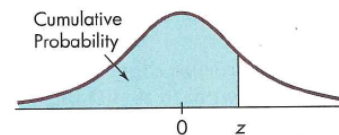
The entries in this table are the cumulative probabilities for the standard normal distribution  $z$  (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a  $z$ -value in the **left-hand tail**.



$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-5.0	0.000003									
-4.5	0.00003									
-4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002
-3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
-3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
-3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
-3.6	0.0002	0.0002	0.0002	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0076	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

**TABLE 3****Cumulative Areas of the Standard Normal Distribution (continued)**

The entries in this table are the cumulative probabilities for the standard normal distribution  $z$  (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a  $z$ -value in the **left-hand tail**.

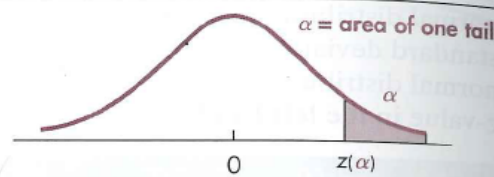


$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.5	0.999997									
5.0	0.9999997									



**TABLE 4****Critical Values of Standard Normal Distribution****A ONE-TAILED SITUATIONS**

The entries in this table are the critical values for  $z$  for which the area under the curve representing  $\alpha$  is in the right-hand tail. Critical values for the left-hand tail are found by symmetry.

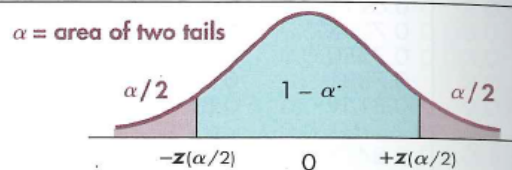


Amount of $\alpha$ in one tail							
$\alpha$	0.25	0.10	0.05	0.025	0.02	0.01	0.005
$z(\alpha)$	0.67	1.28	1.65	1.96	2.05	2.33	2.58

One-tailed example:  
 $\alpha = 0.05$   
 $z(\alpha) = z(0.05) = 1.65$

**B TWO-TAILED SITUATIONS**

The entries in this table are the critical values for  $z$  for which the area under the curve representing  $\alpha$  is split equally between the two tails.



Amount of $\alpha$ in two tails						
$\alpha$	0.25	0.20	0.10	0.05	0.02	0.01
$z(\alpha/2)$	1.15	1.28	1.65	1.96	2.33	2.58
$1 - \alpha$	0.75	0.80	0.90	0.95	0.98	0.99

Two-tailed example:  
 $\alpha = 0.05$  or  $1 - \alpha = 0.95$   
 $\alpha/2 = 0.025$   
 $z(\alpha/2) = z(0.025) = 1.96$

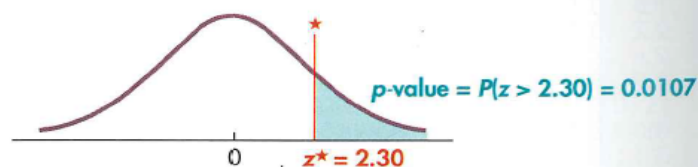
Area in the "center"

For specific details about using Table A to find critical values, see page 393.

For specific details about using Table B to find confidence coefficients, see pages 348, 350, 356; for critical values, see pages 393, 395–396.

**TABLE 5****p-Values for Standard Normal Distribution**

The entries in this table are the  $p$ -values related to the right-hand tail for the calculated  $z^*$  for the standard normal distribution.



$z^*$	$p\text{-value}$	$z^*$	$p\text{-value}$	$z^*$	$p\text{-value}$	$z^*$	$p\text{-value}$	$z^*$	$p\text{-value}$
0.00	0.5000	0.80	0.2119	1.60	0.0548	2.40	0.0082	3.20	0.0007
0.05	0.4801	0.85	0.1977	1.65	0.0495	2.45	0.0071	3.25	0.0006
0.10	0.4602	0.90	0.1841	1.70	0.0446	2.50	0.0062	3.30	0.0005
0.15	0.4404	0.95	0.1711	1.75	0.0401	2.55	0.0054	3.35	0.0004
0.20	0.4207	1.00	0.1587	1.80	0.0359	2.60	0.0047	3.40	0.0003
0.25	0.4013	1.05	0.1469	1.85	0.0322	2.65	0.0040	3.45	0.0003
0.30	0.3821	1.10	0.1357	1.90	0.0287	2.70	0.0035	3.50	0.0002
0.35	0.3632	1.15	0.1251	1.95	0.0256	2.75	0.0030	3.55	0.0002
0.40	0.3446	1.20	0.1151	2.00	0.0228	2.80	0.0026	3.60	0.0002
0.45	0.3264	1.25	0.1056	2.05	0.0202	2.85	0.0022	3.65	0.0001
0.50	0.3085	1.30	0.0968	2.10	0.0179	2.90	0.0019	3.70	0.0001
0.55	0.2912	1.35	0.0885	2.15	0.0158	2.95	0.0016	3.75	0.0001
0.60	0.2743	1.40	0.0808	2.20	0.0139	3.00	0.0013	3.80	0.0001
0.65	0.2578	1.45	0.0735	2.25	0.0122	3.05	0.0011	3.85	0.0001
0.70	0.2420	1.50	0.0668	2.30	0.0107	3.10	0.0010	3.90	0+
0.75	0.2266	1.55	0.0606	2.35	0.0094	3.15	0.0008	3.95	0+



**TABLE 6****Critical Values of Student's *t*-Distribution**

The entries in this table are the critical values of the Student's *t*-distribution, for which the area under the curve is: a) in the right-hand tail, or b) in two tails. See the illustrations at the bottom of the page.

**Area in One Tail**

	0.25	0.10	0.05	0.025	0.01	0.005
<b>Area in Two Tails</b>						
df	0.50	0.20	0.10	0.05	0.02	0.01
3	0.765	1.64	2.35	3.18	4.54	5.84
4	0.741	1.53	2.13	2.78	3.75	4.60
5	0.727	1.48	2.02	2.57	3.36	4.03
6	0.718	1.44	1.94	2.45	3.14	3.71
7	0.711	1.41	1.89	2.36	3.00	3.50
8	0.706	1.40	1.86	2.31	2.90	3.36
9	0.703	1.38	1.83	2.26	2.82	3.25
10	0.700	1.37	1.81	2.23	2.76	3.17
11	0.697	1.36	1.80	2.20	2.72	3.11
12	0.695	1.36	1.78	2.18	2.68	3.05
13	0.694	1.35	1.77	2.16	2.65	3.01
14	0.692	1.35	1.76	2.14	2.62	2.98
15	0.691	1.34	1.75	2.13	2.60	2.95
16	0.690	1.34	1.75	2.12	2.58	2.92
17	0.689	1.33	1.74	2.11	2.57	2.90
18	0.688	1.33	1.73	2.10	2.55	2.88
19	0.688	1.33	1.73	2.09	2.54	2.86
20	0.687	1.33	1.72	2.09	2.53	2.85
21	0.686	1.32	1.72	2.08	2.52	2.83
22	0.686	1.32	1.72	2.07	2.51	2.82
23	0.685	1.32	1.71	2.07	2.50	2.81
24	0.685	1.32	1.71	2.06	2.49	2.80
25	0.684	1.32	1.71	2.06	2.49	2.79
26	0.684	1.31	1.70	2.05	2.47	2.77
27	0.684	1.31	1.70	2.05	2.47	2.77
28	0.683	1.31	1.70	2.05	2.47	2.76
29	0.683	1.31	1.70	2.05	2.46	2.76
30	0.683	1.31	1.70	2.04	2.46	2.75
35	0.682	1.31	1.69	2.03	2.44	2.72
40	0.681	1.30	1.68	2.02	2.42	2.70
50	0.679	1.30	1.68	2.01	2.40	2.68
70	0.678	1.29	1.67	1.99	2.38	2.65
100	0.677	1.29	1.66	1.98	2.36	2.63
df > 100	0.675	1.28	1.65	1.96	2.33	2.58

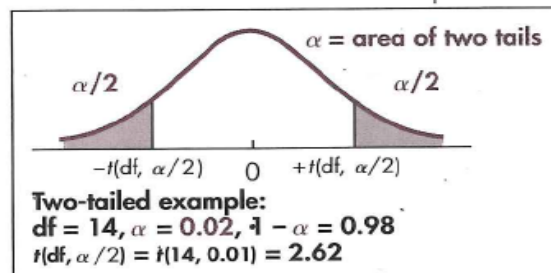
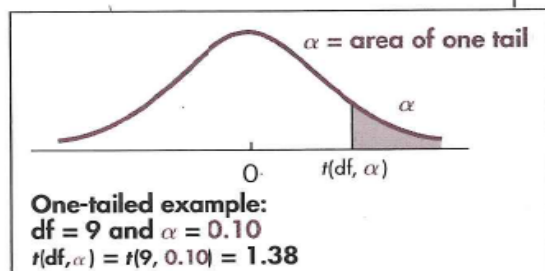
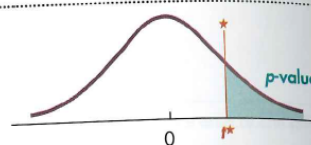


TABLE 7

Probability-Values for Student's *t*-distribution

The entries in this table are the *p*-values related to the right-hand tail for the calculated  $t^*$  value for the *t*-distribution of *df* degrees of freedom.



$t^*$	Degrees of Freedom														
	3	4	5	6	7	8	10	12	15	18	21	25	29	35	$df \geq 45$
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.463	0.463	0.462	0.462	0.462	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461
0.2	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.422	0.422	0.422	0.422	0.422	0.421	0.421	0.421
0.3	0.392	0.390	0.388	0.387	0.386	0.386	0.385	0.385	0.384	0.384	0.384	0.383	0.383	0.383	0.383
0.4	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.348	0.347	0.347	0.347	0.346	0.346	0.346	0.346
0.5	0.326	0.322	0.319	0.317	0.316	0.315	0.314	0.313	0.312	0.312	0.311	0.311	0.310	0.310	0.310
0.6	0.295	0.290	0.287	0.285	0.284	0.283	0.281	0.280	0.279	0.278	0.277	0.277	0.276	0.276	0.276
0.7	0.267	0.261	0.258	0.255	0.253	0.252	0.250	0.249	0.247	0.246	0.246	0.245	0.245	0.244	0.244
0.8	0.241	0.234	0.230	0.227	0.225	0.223	0.221	0.220	0.218	0.217	0.216	0.216	0.215	0.215	0.214
0.9	0.217	0.210	0.205	0.201	0.199	0.197	0.195	0.193	0.191	0.190	0.189	0.188	0.188	0.187	0.186
1.0	0.196	0.187	0.182	0.178	0.175	0.173	0.170	0.169	0.167	0.165	0.164	0.163	0.163	0.162	0.161
1.1	0.176	0.167	0.161	0.157	0.154	0.152	0.149	0.146	0.144	0.143	0.142	0.141	0.140	0.139	0.139
1.2	0.158	0.148	0.142	0.138	0.135	0.132	0.129	0.127	0.124	0.123	0.122	0.121	0.120	0.119	0.118
1.3	0.142	0.132	0.125	0.121	0.117	0.115	0.111	0.109	0.107	0.105	0.104	0.103	0.102	0.101	0.100
1.4	0.128	0.117	0.110	0.106	0.102	0.100	0.096	0.093	0.091	0.089	0.088	0.087	0.086	0.085	0.084
1.5	0.115	0.104	0.097	0.092	0.089	0.086	0.082	0.080	0.077	0.075	0.074	0.073	0.072	0.071	0.070
1.6	0.104	0.092	0.085	0.080	0.077	0.074	0.070	0.068	0.065	0.064	0.062	0.061	0.060	0.059	0.058
1.7	0.094	0.082	0.075	0.070	0.066	0.064	0.060	0.057	0.055	0.053	0.052	0.051	0.050	0.049	0.048
1.8	0.085	0.073	0.066	0.061	0.057	0.055	0.051	0.049	0.046	0.044	0.043	0.042	0.041	0.040	0.039
1.9	0.077	0.065	0.058	0.053	0.050	0.047	0.043	0.041	0.038	0.037	0.036	0.035	0.034	0.033	0.032
2.0	0.070	0.058	0.051	0.046	0.043	0.040	0.037	0.034	0.032	0.030	0.029	0.028	0.027	0.027	0.026
2.1	0.063	0.052	0.045	0.040	0.037	0.034	0.031	0.029	0.027	0.025	0.024	0.023	0.022	0.022	0.021
2.2	0.058	0.046	0.040	0.035	0.032	0.029	0.026	0.024	0.022	0.021	0.020	0.019	0.018	0.017	0.016
2.3	0.052	0.041	0.035	0.031	0.027	0.025	0.022	0.020	0.018	0.017	0.016	0.015	0.014	0.014	0.013
2.4	0.048	0.037	0.031	0.027	0.024	0.022	0.019	0.017	0.015	0.014	0.013	0.012	0.012	0.011	0.010
2.5	0.044	0.033	0.027	0.023	0.020	0.018	0.016	0.014	0.012	0.011	0.010	0.010	0.009	0.009	0.008
2.6	0.040	0.030	0.024	0.020	0.018	0.016	0.013	0.012	0.010	0.009	0.008	0.008	0.007	0.007	0.006
2.7	0.037	0.027	0.021	0.018	0.015	0.014	0.011	0.010	0.008	0.007	0.007	0.006	0.006	0.005	0.005
2.8	0.034	0.024	0.019	0.016	0.013	0.012	0.009	0.008	0.007	0.006	0.005	0.005	0.005	0.004	0.004
2.9	0.031	0.022	0.017	0.014	0.011	0.010	0.008	0.007	0.005	0.005	0.004	0.004	0.004	0.003	0.003
3.0	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.006	0.004	0.004	0.003	0.003	0.003	0.002	0.002
3.1	0.027	0.018	0.013	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.001
3.2	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001
3.3	0.023	0.015	0.011	0.008	0.007	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001
3.4	0.021	0.014	0.010	0.007	0.006	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.5	0.020	0.012	0.009	0.006	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
3.6	0.018	0.011	0.008	0.006	0.004	0.004	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.7	0.017	0.010	0.007	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.8	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.9	0.015	0.009	0.006	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4.0	0.014	0.008	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001



# Formula Card for Johnson & Kuby, ELEMENTARY STATISTICS, Eleventh Edition

**Sample mean:**

$$\bar{x} = \frac{\sum x}{n} \quad (2.1)$$

**Depth of sample median:**

$$d(\bar{x}) = (n + 1)/2 \quad (2.2)$$

**Range:**  $H - L$

**Sample variance:**

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1} \quad (2.5)$$

or

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} \quad (2.9)$$

**Sample standard deviation:**

$$s = \sqrt{s^2} \quad (2.6)$$

**Chebyshev's theorem:** at least  $1 - (1/k^2)$

(p. 99)

**Sum of squares of  $x$ :**

$$SS(x) = \sum x^2 - ((\sum x)^2/n) \quad (2.8)$$

**Sum of squares of  $y$ :**

$$SS(y) = \sum y^2 - ((\sum y)^2/n) \quad (3.3)$$

**Sum of squares of  $xy$ :**

$$SS(xy) = \sum xy - ((\sum x \cdot \sum y)/n) \quad (3.4)$$

**Pearson's correlation coefficient:**

$$r = SS(xy) / \sqrt{SS(x) \cdot SS(y)} \quad (3.2)$$

**Equation for line of best fit:**  $\hat{y} = b_0 + b_1x$

(p. 146)

**Slope for line of best fit:**  $b_1 = SS(xy)/SS(x)$

(3.6)

**y-intercept for line of best fit:**

$$b_0 = [\sum y - (b_1 \cdot \sum x)]/n \quad (3.7)$$

**Empirical (observed) probability:**

$$P'(A) = n(A)/n \quad (4.1)$$

**Theoretical probability for equally likely sample space:**

$$P(A) = n(A)/n(S) \quad (4.2)$$

**Complement rule:**

$$P(\text{not } A) = P(\bar{A}) = 1 - P(A) \quad (4.3)$$

**General addition rule:**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (4.4)$$

**General multiplication rule:**

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad (4.5)$$

**Special addition rule for mutually exclusive events:**

$$P(A \text{ or } B \text{ or } \dots \text{ or } E) = P(A) + P(B) + \dots + P(E) \quad (4.6)$$

**Special multiplication rule for independent events:**

$$P(A \text{ and } B \text{ and } \dots \text{ and } E) = P(A) \cdot P(B) \cdot \dots \cdot P(E) \quad (4.7)$$

**Mean of discrete random variable:**

$$\mu = \sum [xP(x)] \quad (5.1)$$

**Variance of discrete random variable:**

$$\sigma^2 = \sum [x^2P(x)] - [\sum [xP(x)]]^2 \quad (5.3a)$$

**Standard deviation of discrete random variable:**

$$\sigma = \sqrt{\sigma^2} \quad (5.4)$$

**Factorial:**  $n! = (n)(n-1)(n-2) \cdot \dots \cdot 2 \cdot 1$  (p. 248)

**Binomial coefficient:**

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad (5.6)$$

**Binomial probability function:**

$$P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}, x = 0, 1, 2, \dots, n \quad (5.5)$$

**Mean of binomial random variable:**  $\mu = np$  (5.7)

**Standard deviation, binomial random variable:**

$$\sigma = \sqrt{npq} \quad (5.8)$$

**Standard score:**  $z = (x - \mu)/\sigma$  (6.3)

**Standard score for  $\bar{x}$ :**  $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$  (7.2)

**Confidence interval for mean,  $\mu$  ( $\sigma$  known):**

$$\bar{x} \pm z_{(\alpha/2)} \cdot (\sigma/\sqrt{n}) \quad (8.1)$$

**Sample size for  $1 - \alpha$  confidence estimate for  $\mu$ :**

$$n = [z_{(\alpha/2)} \cdot \sigma/E]^2 \quad (8.3)$$

**Calculated test statistic for  $H_0: \mu = \mu_0$  ( $\sigma$  known):**

$$z^* = (\bar{x} - \mu_0)/(\sigma/\sqrt{n}) \quad (8.4)$$

**Confidence interval estimate for mean,  $\mu$**

**( $\sigma$  unknown):**

$$\bar{x} \pm t_{(df, \alpha/2)} \cdot (s/\sqrt{n}) \text{ with } df = n - 1 \quad (9.1)$$

**Calculated test statistic for  $H_0: \mu = \mu_0$  ( $\sigma$  unknown):**

$$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ with } df = n - 1 \quad (9.2)$$

**Confidence interval estimate for proportion,  $p$ :**

$$p' \pm z_{(\alpha/2)} \cdot \sqrt{(p'q')/n}, p' = x/n \quad (9.6)$$

**Calculated test statistic for  $H_0: p = p_0$ :**

$$z^* = (p' - p_0)/\sqrt{(p_0q_0/n)}, p' = x/n \quad (9.9)$$

**Calculated test statistic for  $H_0: \sigma^2 = \sigma_0^2$  or  $\sigma = \sigma_0$ :**

$$\chi^2 = (n-1)s^2/\sigma_0^2, df = n - 1 \quad (9.10)$$

**Mean difference between two dependent samples:**

**Paired difference:**  $d = x_1 - x_2$  (10.1)

**Confidence interval for mean difference,  $\mu_d$ :**

$$\bar{d} \pm t_{(df, \alpha/2)} \cdot s_d/\sqrt{n} \text{ with } df = n - 1 \quad (10.2)$$

**Sample mean of paired differences:**

$$\bar{d} = \sum d/n \quad (10.3)$$

Sample standard deviation of paired differences:

$$s_d = \sqrt{\frac{\sum d^2 - (\sum d)^2/n}{n-1}} \quad (10.4)$$

Calculated test statistic for  $H_0: \mu_d = \mu_0$ :

$$t^* = (\bar{d} - \mu_0) / (s_d / \sqrt{n}), \quad df = n - 1 \quad (10.5)$$

Difference between means of two independent samples:

Degrees of freedom:

$$df = \text{smaller of } (n_1 - 1) \text{ or } (n_2 - 1) \quad (\text{p. 496})$$

Confidence interval estimate for  $\mu_1 - \mu_2$ :

$$(\bar{x}_1 - \bar{x}_2) \pm t_{(df, \alpha/2)} \sqrt{(s_1^2/n_1) + (s_2^2/n_2)} \quad (10.8)$$

Calculated test statistic for  $H_0: \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$ :

$$t^* = [(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0] / \sqrt{(s_1^2/n_1) + (s_2^2/n_2)} \quad (10.9)$$

Difference between proportions of two independent samples:

Confidence interval for  $p_1 - p_2$ :

$$(p'_1 - p'_2) \pm z_{(\alpha/2)} \cdot \sqrt{\frac{p'_1 q'_1}{n_1} + \frac{p'_2 q'_2}{n_2}} \quad (10.11)$$

Pooled observed probability:

$$p'_p = (x_1 + x_2) / (n_1 + n_2) \quad (10.13)$$

$$q'_p = 1 - p'_p \quad (10.14)$$

Calculated test statistic for  $H_0: p_1 - p_2 = 0$ :

$$z^* = \frac{p'_1 - p'_2}{\sqrt{(p'_p)(q'_p) \left[ \left( \frac{1}{n_1} + \frac{1}{n_2} \right) \right]}} \quad (10.15)$$

Ratio of variances between two independent samples:

Calculated test statistic for  $H_0: \sigma_1^2/\sigma_2^2 = 1$ :

$$F^* = s_1^2/s_2^2 \quad (10.16)$$

Calculated test statistic for enumerative data:

$$\chi^2 = \sum [(O - E)^2/E] \quad (11.1)$$

Multinomial experiment:

Degrees of freedom:  $df = k - 1$

Expected frequency:  $E = n \cdot p$

Test for independence or Test of homogeneity:

Degrees of freedom:

$$df = (r - 1) \cdot (c - 1) \quad (11.4)$$

$$\text{Expected value: } E = (R \cdot C) / n \quad (11.5)$$

Mathematical model:

$$x_{c,k} = \mu + F_c + \epsilon_{k(c)} \quad (12.13)$$

Total sum of squares:

$$SS(\text{total}) = \sum (x^2) - \frac{(\sum x)^2}{n} \quad (12.2)$$

Sum of squares due to factor:

$$\left[ \left( \frac{C_1^2}{k_1} \right) + \left( \frac{C_2^2}{k_2} \right) + \left( \frac{C_3^2}{k_3} \right) + \dots \right] - \left[ \frac{(\sum x)^2}{n} \right] \quad (12.3)$$

Sum of squares due to error:

$$SS(\text{error}) = \sum (x^2) - [(C_1^2/k_1) + (C_2^2/k_2) + (C_3^2/k_3) + \dots] \quad (12.4)$$

Degrees of freedom for total:

$$df(\text{total}) = n - 1 \quad (12.6)$$

Degrees of freedom for factor:

$$df(\text{factor}) = c - 1 \quad (12.5)$$

Degrees of freedom for error:

$$df(\text{error}) = n - c \quad (12.7)$$

Mean square for factor:

$$MS(\text{factor}) = SS(\text{factor}) / df(\text{factor}) \quad (12.10)$$

Mean square for error:

$$MS(\text{error}) = SS(\text{error}) / df(\text{error}) \quad (12.11)$$

Calculated test statistic for  $H_0$ : Mean value is same at all levels:

$$F^* = MS(\text{factor}) / MS(\text{error}) \quad (12.12)$$

Covariance of  $x$  and  $y$ :

$$\text{covar}(x, y) = \sum [(x - \bar{x})(y - \bar{y})] / (n - 1) \quad (13.1)$$

Pearson's correlation coefficient:

$$r = \text{covar}(x, y) / (s_x \cdot s_y) \quad (13.2)$$

or

$$r = SS(xy) / \sqrt{SS(x) \cdot SS(y)} \quad (3.2) \text{ or } (13.3)$$

$$\text{Experimental error: } e = y - \hat{y} \quad (13.5)$$

$$\text{Estimated variance of error: } s_e^2 = \sum (y - \hat{y})^2 / (n - 2) \quad (13.6)$$

or

$$s_e^2 = \frac{(\sum y^2) - (b_0)(\sum y) - (b_1)(\sum xy)}{n - 2} \quad (13.8)$$

Standard deviation about the line of best fit:

$$s_e = \sqrt{s_e^2} \quad (13.9)$$

Estimate for variance of slope:

$$s_{b_1}^2 = \frac{s_e^2}{SS(x)} = \frac{s_e^2}{\sum x^2 - [(\sum x)^2/n]} \quad (13.12)$$

Confidence interval for  $\beta_1$ :

$$b_1 \pm t(df, \alpha/2) \cdot s_{b_1} \quad (13.14)$$

Calculated test statistic for  $H_0: \beta_1 = 0$ :

$$t^* = (b_1 - \beta_1) / s_{b_1} \text{ with } df = n - 2 \quad (13.15)$$

Confidence interval for mean value of  $y$  at  $x_0$ :

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS(x)}} \quad (13.17)$$

Prediction interval for  $y$  at  $x_0$ :

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS(x)}} \quad (13.16)$$

Mann-Whitney  $U$  test:

$$U_a = n_a \cdot n_b + [(n_b) \cdot (n_b + 1) / 2] - R_b \quad (14.3)$$

$$U_b = n_a \cdot n_b + [(n_a) \cdot (n_a + 1) / 2] - R_a \quad (14.4)$$

Spearman's rank correlation coefficient:

$$r_s = 1 - \left[ \frac{6 \sum d^2}{n(n^2 - 1)} \right] \quad (14.11)$$